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Comparative analysis of passive algorithms in adaptive control for a specific application.

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***Abstract*—Passivity based adaptive control has had a wide variety of practical applications due to its robustness, flexibility and modularity. There exists representative adaptive algorithms (AAs) based on this approach such as Gradient (G) (Ortega, Spong, 1989), Composite (C) (Slotine, Li, 1989), Average (A) (Tang, Arteaga, 1994), and Modified Least Squares (MLS) (Lozano, Canudas, 1990). Comparison[1] among them has not been made. Proportional-integral (PI) type AAs developed in references (Astolfi, Ortega, 2003; Qu, et.al., 2006; Tyukin et. al., 2007; Wang, Slotine, 2006) are attractive due to their ability to handle nonlinear parameters. Nevertheless, passivity properties for PI algorithms have not been explored yet. In this paper we present a general unified passive scheme that encompasses all aforementioned AAs including PI (for linear parameterization). A comparative analysis in performances among them was carried out considering the problem of passivity-based adaptive control of a simple pendulum and tracking of two trajectories.**

Key words: Passive elements, Adaptive Algorithms, Stability.[2]

I. Introduction

In the early 1970's, passivity and dissipativity concepts were introduced by Willems (Willems, 1972) and the notions of storage functions and supply rate were established. Interconnections of passive elements have particular features which were used by adaptive control theory to generate the so called passivity based adaptive control, which is nowadays a widely applied methodology of control. Representative passive AAs have been developed such as Gradient (G) (Ortega, Spong, 1989), Composite (C) (Slotine, Li, 1989), Average (A) (Tang, Arteaga, 1994), and Modified Least Squares (MLS) (Lozano, Canudas, 1990) but comparison among them has not been studied. In another hand, PI type AAs recently reported in (Astolfi, Ortega, 2003; Qu, et. al., 2006; Tyukin, et. al., 2007; Wang, Slotine, 2006), introduce suitable degrees of freedom which can be used to handle nonlinear parameters but, to the best of our knowledge, passivity properties for such schemes have not been explored. Since each algorithm was developed by independent reports, they are apparently related only by the fact that they are passive. Nevertheless, in this report we present a general passive adaptive law that encompasses all aforementioned algorithms, including PI for linear parameterization. It is worth nothing that not all particular cases of a passive algorithm are passive, for example, MLS scheme (Lozano, Canudas, 1990) is passive but its particular case, LS type adaptive law (Slotine, Li, 1991), is not. Since all aforementioned AAs comes from the same general scheme, the natural question to ask is, which one has "better" performance for a specific application?. In this article it is proposed a partial answer by giving a comparative analysis in performances among all AAs, considering the model of simple pendulum and tracking control.

[1]As far as we know

[2]The work is supported by CONACyT No. 129800 and PAPIIT No. 116412, México

II. Passive Adaptive Algorithms

A. Linear error dynamics

The equation that describes the movement of a simple pendulum is:

$$ml^2\ddot{y} + kl^2\dot{y} + mgl\sin(y) = u \triangleq Y(\ddot{y},\dot{y},y)\theta, \quad (1)$$

with mass m, length l, acceleration due to the gravity g, friction coefficient k and control input (torque) u. Variable y denotes angle and its time derivatives are represented with appropriate number of dots. $Y(\ddot{y},\dot{y},y) \triangleq \begin{pmatrix} \ddot{y} & \dot{y} & \sin(y) \end{pmatrix}^T$ and $\theta \triangleq \begin{pmatrix} ml^2 & kl^2 & mgl \end{pmatrix}^T$ is the real parameters vector. Defining the tracking error variable as $e = y(t) - y_d(t)$, where $y_d(t)$ a twice differentiable known path, the sliding variable as

$$s = \dot{e} + \lambda e, \ \lambda > 0, \quad (2)$$

and time derivative of the reference variable as $\dot{y}_r(t) = \dot{y}_d(t) - \lambda e$, equation (1) can be written as:

$$ml^2\dot{s} = u - \left(ml^2\ddot{y}_r + kl^2\dot{y} + mgl\sin(y)\right) \triangleq u - \varphi^T\theta, \quad (3)$$

where $\varphi = \begin{pmatrix} \ddot{y}_r & \dot{y} & \sin(y) \end{pmatrix}^T$ is called regressor. Adaptive control expression given by:

$$u_a = -k_s s + \varphi^T\hat{\theta}, \ k_s > 0, \quad (4)$$

renders the closed loop with (3) as:

$$ml^2\dot{s} = -k_s s + \varphi^T\tilde{\theta}, \quad (5)$$

where $\tilde{\theta} = \hat{\theta} - \theta$. In order to establish stability properties for closed loop system (5), it is proposed the positive definite function $V_s(s, \tilde{\theta}) = \frac{1}{2} m l^2 s^2 + \frac{\gamma_1}{2} \tilde{\theta}^T \tilde{\theta}$. Its time derivative along (5) gives $\dot{V}_s(s, \tilde{\theta}) = -k_s s^2$ where the AA, $\dot{\hat{\theta}} = -\gamma_1 \varphi s$ (Gradient (G) algorithm), was chosen. Then $s \in L_2 \cap L_\infty$. Since $\tilde{\theta} \in L_\infty$ then $\dot{s} \in L_\infty$ (by (5)) and, by Barbalat's lemma, $s \to 0$ as $t \to \infty$. This problem has a more general approach (see (Ortega, Spong, 1989)) if the system (5) is connected on a feedback configuration with *any* passive adaptive law, as is depicted in Fig. (1). Let us assume that AA in block H_2 is passive from

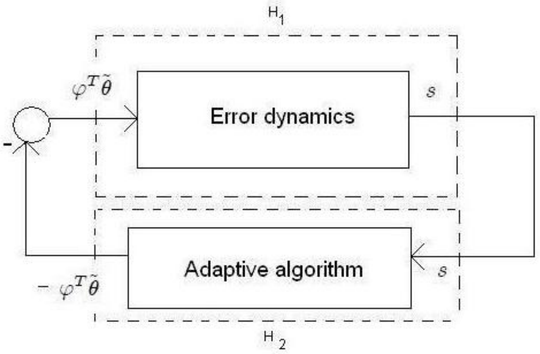

Figure 1. Feedback connection of error dynamics, equation (5), in H_1 block, and any passive AA in block H_2.

$s \to -\varphi^T \tilde{\theta}$ and $\tilde{\theta} \in L_\infty$. Then, H_2 satisfies the following inequality:

$$-\int_0^T s \varphi^T \tilde{\theta} dt \geq -\beta, \ \forall T \geq 0, \ \beta \geq 0. \quad (6)$$

Now, it is possible to state the following lemma (see (Ortega, Spong, 1989)).

Lemma 1: Consider tracking error dynamics (5) is connected in H_1 block of Fig. (1) and *any* AA which satisfies (6) is connected in H_2 block. In addition, assume $\tilde{\theta} \in L_\infty$. Then $s \to 0$ as $t \to \infty$.

Proof: The positive definite function:

$$V(s, \tilde{\theta}) = \frac{1}{2} m l^2 s^2 + \beta - \int_0^t s \varphi^T \tilde{\theta} d\tau > 0, \ \forall s \neq 0$$

has time derivative along (5) as:

$$\dot{V}(s, \tilde{\theta}) = -k_s s^2 \leq 0.$$

Then $s \in L_2 \cap L_\infty$. Since $\tilde{\theta} \in L_\infty$, then $\dot{s} \in L_\infty$ (by (5)) and by Barbalat's lemma, $s \to 0$ as $t \to \infty$. ■

As a result, any passive AA connected in H_2 block will render the point $s = 0$ asymptotically stable.

If the estimation error variable $\tilde{\theta}(t)$ and the AA are generalized, it is possible to demonstrate that all representative passive adaptive laws are encompassed in an unified scheme. In addition, it is possible to demonstrate passivity properties of PI-AAs (Astolfi, Ortega, 2003; Tyukin, et. al., 2007; Qu, et. al., 2006; Wang, Slotine, 2006) in case of linear parameterization.

Consider estimation variable as $\hat{\theta} = \hat{\theta}_I(t) + \hat{\theta}_P(s)$ as in references (Astolfi, Ortega, 2003; Qu, et. al., 2006; Tyukin, et. al., 2007; Wang, Slotine, 2006). $\hat{\theta}_I(t)$ is related to conventional Lyapunov-based designs and $\hat{\theta}_P(s)$ is considered a new degree of freedom which introduce an interaction between tracking error s and estimation error dynamics. Defining the estimation error variable as $\tilde{\theta} = \hat{\theta}_I - \theta + \hat{\theta}_P$ and estimation error terms as $\tilde{\theta}_I(t) = \hat{\theta}_I(t) - \theta$ and $\tilde{\theta}_P(s) = \hat{\theta}_P(s)$, the following general passive AA encompasses the mentioned representative algorithms.

Proposition 2: Consider the following adaptive algorithm:

$$\begin{aligned} \dot{\tilde{\theta}}_I(t) &= -\delta Z(t) \tilde{\theta}_I(t) - \gamma_1 P(t) \varphi s, \quad (7) \\ -\gamma_1 \tilde{\theta}_P^T(s) \varphi s &\geq \frac{1}{2} \tilde{\theta}_I^T \frac{dP(t)^{-1}}{dt} \tilde{\theta}_I. \quad (8) \end{aligned}$$

If $\theta \in \mathbb{R}^m$, then $\hat{\theta}_I(t) : \mathbb{R}_+ \to \mathbb{R}^m$ and $\hat{\theta}_P(s) : \mathbb{R} \to \mathbb{R}^m$. The general matrix function $Z(t) : \mathbb{R}_+ \to \mathbb{R}^{m \times m}$ satisfies $Z(t) \geq 0$ for all $t \geq 0$, the time varying matrix gain $P(t) : \mathbb{R}_+ \to \mathbb{R}^{m \times m}$ is such that $P(t) > 0$ for all $t \geq 0$. Real constants δ and γ_1 are strictly positive. With all aforementioned conditions, algorithm (7)-(8) is passive from $s \to -\varphi_T \tilde{\theta}$.

Proof: The positive definite function $V(t) = \frac{1}{2\gamma_1} \tilde{\theta}_I^T P(t)^{-1} \tilde{\theta}_I$ has time derivative along (7) given by:

$$\begin{aligned} \dot{V(t)} &= \frac{1}{\gamma_1} \tilde{\theta}_I^T P^{-1} \dot{\tilde{\theta}}_I + \frac{1}{2\gamma_1} \tilde{\theta}_I^T \frac{dP^{-1}}{dt} \tilde{\theta}_I \\ &\leq \tilde{\theta}_I^T P^{-1} \left(-P \varphi s - \frac{\delta}{\gamma_1} Z \tilde{\theta}_I \right) - \tilde{\theta}_P^T \varphi s, \\ &= -\tilde{\theta}_I^T \varphi s - \frac{\delta}{\gamma_1} \tilde{\theta}_I^T P^{-1} Z \tilde{\theta}_I - \tilde{\theta}_P^T \varphi s \\ &= -\left(\tilde{\theta}_I + \tilde{\theta}_P \right)^T \varphi s - \frac{\delta}{\gamma_1} \tilde{\theta}_I^T P^{-1} Z \tilde{\theta}_I \\ &\leq -\tilde{\theta}^T \varphi s. \end{aligned}$$

Equation (8) was used in the second line. Then, (7)-(8) is passive from $s \to -\tilde{\theta}^T \varphi$. ■

From adaptive passive law (7)-(8), two groups of adaptive laws can be differentiated: one in which $\delta > 0$, $\gamma_1 > 0$, $P(t) = 1$, $\hat{\theta}_P(s) = 0$ and as a result, estimated parameters vector $\hat{\theta}$ has only $\hat{\theta}_I(t)$ term. In this case equation (8) is satisfied because $0 = \tilde{\theta}_I^T \frac{dP(t)}{dt}^{-1} \tilde{\theta}_I \leq -\gamma_1 \tilde{\theta}_P^T \varphi s = 0$. Within this group of algorithms they are found:

- Gradient Algorithm (Ortega, Spong, 1989).

$$Z(t) = 0, \quad (9)$$

which results in the expression $\dot{\hat{\theta}}_I = \dot{\tilde{\theta}}_I = -\gamma_1 \varphi s$.

- Composite (with fixed gains) Algorithm (Slotine, Li, 1989).

$$\begin{aligned} Y_f(\dot{y}, y) &= W(p)Y(\ddot{y}, \dot{y}, y), \\ W(p) &= \frac{\lambda_f}{p+\lambda_f}, \\ Z(t) &= Y_f Y_f^T, \end{aligned} \quad (10)$$

where $Y(\ddot{y}, \dot{y}, y)$ is defined in equation (1) and p is the Laplace operator. Adaptive law in this case is (in analysis form) $\dot{\tilde{\theta}}_I = -\delta Y_f Y_f^T \tilde{\theta}_I - \gamma_1 \varphi s$ or (in implementable form) $\dot{\hat{\theta}}_I = -\delta Y_f \epsilon(t) - \gamma_1 \varphi s$ where $\epsilon(t) = Y_f^T \hat{\theta}_I - u_f$ and $u_f = W(p)u$, u as in equation (1).

- Average Algorithm (Tang, Arteaga, 1994).

$$\begin{aligned} Z(t) &= \gamma_2 \int_0^t \exp(-\lambda_c(t-\tau)) Y_f Y_f^T d\tau, \\ Y_f(\dot{y}, y) &= W(p)Y(\ddot{y}, \dot{y}, y), \\ W(p) &= \frac{\lambda_f}{p+\lambda_f}, \\ \lambda_c &\geq 0, \ \gamma_2 > 0. \end{aligned}$$

Adaptive law can be expressed as (see (Tang, Arteaga, 1994))

$$\dot{\tilde{\theta}}_I(t) = -\delta Z(t)\tilde{\theta}_I(t) - \gamma_1 \varphi s. \quad (11)$$

In implementable form:

$$\dot{\hat{\theta}}_I(t) = -\delta g(t) - \gamma_1 \varphi s,$$

where

$$\begin{aligned} \dot{g}(t) &= -(\lambda_c I + \delta R(t))\, g(t) + \gamma_2 Y_f \epsilon(t) \\ &\quad -\gamma_1 R(t) \varphi s, \\ \dot{R}(t) &= -\lambda_c R(t) + \gamma_2 Y_f Y_f^T, \\ g(0) &= 0, \ R(0) \ = \ 0. \end{aligned}$$

In another hand, the second group satisfies $\hat{\theta}_P(s) \neq 0$ then, vector of estimated parameters is $\hat{\theta} = \hat{\theta}_I(t) + \hat{\theta}_P(s)$. For the following cases $\delta > 0$, $\gamma_1 > 0$, $Z(t) = 0$, in PI case $P(t)$ is constant, and in MLS case $P(t)$ is time varying. Different choices of $\tilde{\theta}_P$ are explained.

- PI algorithm (Astolfi, Ortega, 2003)

$$\begin{aligned} \dot{\tilde{\theta}}_I(t) &= -\gamma_1 P(t) \varphi s, & (12) \\ \tilde{\theta}_P(s) &= -\gamma_P \varphi s, & (13) \end{aligned}$$

where $\gamma_P = \gamma_1 m l^2$, $\gamma_1 > 0$, $P(t) = k_s$ where k_s is constant defined in equation (5). Condition (8) is satisfied because $0 = \tilde{\theta}_I^T \frac{dP(t)}{dt}^{-1} \tilde{\theta}_I \leq -\gamma_1 \tilde{\theta}_P(s)^T \varphi s = \gamma_1 \gamma_P \varphi^T \varphi s^2$ with $\tilde{\theta}_P$ as in (13). Equations (12)-(13) gives the expression of time derivative of estimation error variable as $\dot{\tilde{\theta}} = -\gamma_1 \varphi \varphi^T \tilde{\theta}$.

- MLS scheme (Lozano, Canudas, 1990). Under the assumption that an upperbound $M \geq \|\theta^*\|^2$ is known, we have:

$$\begin{aligned} \dot{\tilde{\theta}}_I &= -P(t) \varphi s, & (14) \\ \hat{\theta}_P &= \frac{-\varphi s \left(\hat{\theta}_I^T A \hat{\theta}_I + M(1 + \lambda_L \cdot \lambda_{\max} R) \right)}{(1 + tr(\varphi \varphi^T))(1+s^2)} \end{aligned}$$

The following selections are picked out in order to render gain $P(t)$ positive definite for all $t \geq 0$ (see (Lozano, Canudas, 1990)):

$$\begin{aligned} \dot{P}(t) &= \alpha(t)(-PAP + \lambda_L P), \\ \alpha(t) &= \frac{\varphi^T \varphi s^2}{(1 + tr(\varphi \varphi^T))(1+s^2)}, \\ A &= \frac{\varphi \varphi^T}{(1 + tr(\varphi \varphi^T))} + \lambda_L R, \\ \lambda_L &\geq 0, \ R > 0, \\ (\lambda_{\min} R)\, I &\leq P^{-1}(0) \leq \left(\lambda_{\max} R + \frac{1}{\lambda_L} \right) I, \end{aligned}$$

where R is a $m \times m$ constant matrix and $\alpha(t) : \mathbb{R}_+ \to \mathbb{R}_+$. It is worth nothing that the above algorithm is exactly the LS algorithm (Slotine, Li, 1991) if $\alpha(t) = 1$, $R = 0$, $\lambda_L = 0$, and $\tilde{\theta}_P = 0$, nevertheless, LS algorithm is not passive. Condition (8) is satisfied because (see (Lozano, Canudas, 1990)):

$$\begin{aligned} & \frac{1}{2}(\hat{\theta}_I - \theta^*)^T \frac{dP^{-1}}{dt} (\hat{\theta}_I - \theta^*) \\ \leq \ & \frac{1}{2}\alpha(t)(\hat{\theta}_I - \theta^*)^T A (\hat{\theta}_I - \theta^*) \\ \leq \ & \alpha(t) \left(\hat{\theta}_I^T A \hat{\theta}_I + \theta^{*T} A \theta^* \right) \\ \leq \ & \alpha(t) \left(\hat{\theta}_I^T A \hat{\theta}_I + M(1 + \lambda \cdot \lambda_{\max} R) \right), \end{aligned}$$

and we obtain:

$$\frac{1}{2}(\hat{\theta}_I - \theta^{*T}) \frac{dP^{-1}}{dt} (\hat{\theta}_I - \theta^*) \leq -\hat{\theta}_P^T \varphi s.$$

B. General error dynamics

A more general version of dynamics (5) is given by:

$$\dot{e} = -\psi(e) + \varphi^T \tilde{\theta}, \quad (15)$$

where e is tracking error variable and $\psi(\cdot)$ is a continuous scalar function which satisfies, $e\psi(e) > 0$, $\forall e \neq 0$ and $\lim_{e \to \infty} \int_0^e \psi(\tau) d\tau \ \to \ \infty$. Result in Lemma 1 holds for this dynamics as can be stated in the following proposition.

Proposition 3: Consider equation (15) is connected in H_1 block of Figure (1) and all conditions of Lemma 1 hold. Then $e \to 0$ as $t \to \infty$.

Proof: Positive definite function $V(e, \tilde{\theta}) = \frac{1}{2}e^2 + \beta - \int_0^T e \varphi^T \tilde{\theta} dt$, has time derivative along trajectories of (15) as:

$$\dot{V}(e, \tilde{\theta}) = -e\psi(e) \leq 0,$$

Table I
PARAMETERS OF SIMULATION FOR EACH PASSIVE AA. IN THE UPPER ROW THERE ARE THE NOMINAL VALUES FOR MASS, FRICTION COEFFICIENT AND LENGTH.

AA	$m^* = 1kg$	$k^* = .015$	$l^* = 1m$
G	$\gamma_1 = 5$		
C	$\gamma_1 = 5$	$\lambda_f = 1$	$\delta = 1$
A	$\gamma_1 = 5$ $\gamma_2 = 10$	$\lambda_f = 1$ $\delta = 1$	$\lambda_c = .05$
PI	$\gamma_1 k_s = 5$	$\gamma_P = 1$	
MLS	$M = 100$	$\lambda_L = 1$ $\lambda_{\max} = 1$	$R = I$ $P(0) = 100 \times I$

Table II
PERFORMANCE PARAMETERS FOR EACH ALGORITHM: SETTLING TIMES FOR CONSTANT AND TIME VARYING REFERENCES (C-ST AND V-ST RESPECTIVELY), AND RMS ERROR FOR CONSTANT AND VARYING REFERENCE (C-RMSE AND V-RMSE RESPECTIVELY).

AA	**C-St [s]**	**V-St [s]**	**C-RMSe**	**V-RMSe**
G	22.3344	25.7691	0.1449	0.1302
C	10.6433	13.9012	0.1141	0.1202
A	6.4681	14.7169	0.0429	0.0699
PI	22.9186	26.2890	0.1391	0.1177
MLS	12.1418	12.8840	0.0072	0.0293

by the same reasons as in Lemma 1, then $e \to 0$ as $t \to \infty$. ■

III. SIMULATIONS AND RESULTS.

Simulations were carried out over model (1), control law (4) and adaptive schemes Gradient, Composite, Average, PI (12)-(13), and MLS (14). Also two trajectories were proved, one time varying, given by $y_d(t) = 1 + \sin(t)$ and the constant path $y_d(t) = \frac{\pi}{4}$. Constants that were used in the simulation, are given in Table I.

The tuning methodology was: beginning with gradient algorithm, a first value of the gain γ_1 which generates qualitatively good response was fixed. Then, composite law has the same γ_1 gain and δ is chosen such that generates qualitative good response, and so on.

For all simulations $k_s = 8$, $\lambda = 1$ in equations (5) and (2) respectively and when part of estimation had to be integrated, it was used the vector $\hat{\theta}(0) = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}^T$ as initial condition.

Once all simulations were carried out, it was used the Matlab command "stepinfo" over the vectors describing tracking errors and the parameter chosen as performance index was settling time (St). As RMS error, the formula $\sqrt{\frac{1}{n}\sum_{i=1}^{i=n} e_i^2}$ was applied.

Table II presents the results for each adaptive law, where C and V denote constant reference and time varying reference respectively; unit of time is second [s]. In Fig.(2) it is depicted the behavior of each algorithm for tracking error and control signal when reference is the time varying trajectory $y_d(t) = 1 + \sin(t)$. Control signals are almost the same for all algorithms. MLS scheme has the smoothest behavior in tracking

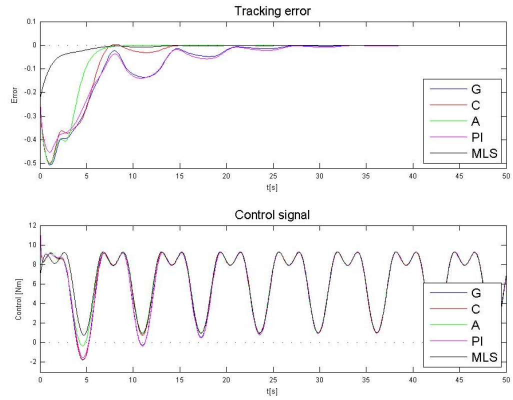

Figure 2. Tracking error and control signal for each one of the five passive AAs in closed loop with (1) and (4). Trajectory of reference is $y_d = 1 + sin(t)$.

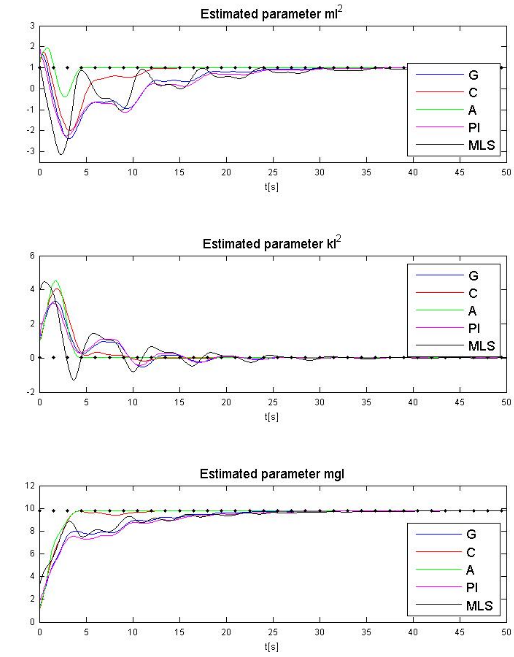

Figure 3. Estimated parameters' behavior with time varying reference $y_d = 1 + sin(t)$.

error but a big overshoot in control[3]. MLS has also better settling time and the smallest RMS error, as can be concluded from Table II. For tracking error, PI has almost the same behavior as Gradient but PI has the biggest settling time (Table II). Due to the presence of persistent excitation through reference signal, all estimated parameters reach the real values (Figure (3)).

[3]It is not depicted in the figure due to comparative purposes. Overshoot magnitude is about 75 units

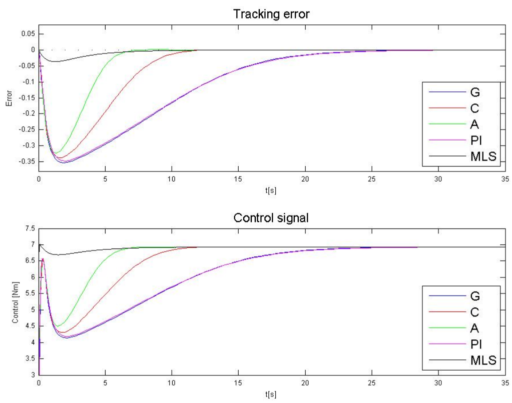

Figure 4. Tracking error and control signal for each one of the five passive AAs in closed loop with (1) and (4). Trajectory of reference is $y_d = \frac{\pi}{4}$.

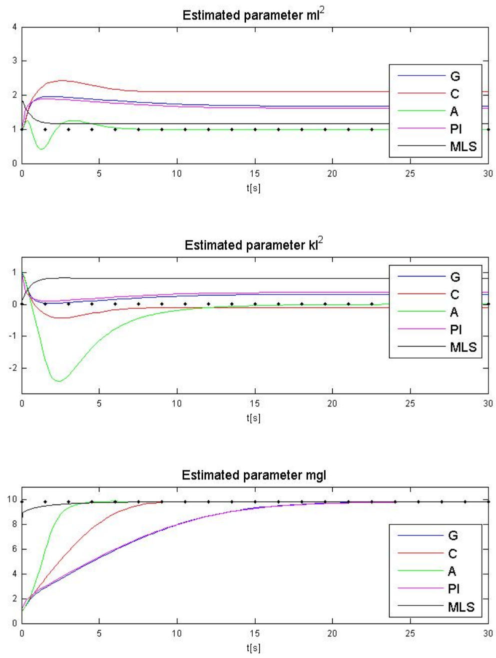

Figure 5. Estimated parameters' behavior with constant reference $y_d = \frac{\pi}{4}$.

The constant reference trajectory $y_d = \frac{\pi}{4}$ was also proved over system (1), adaptive control (4) and all passive algorithms. Tracking error and control signals are depicted in Figure (4). MLS has the smoothest behavior in tracking error and control signal[4]. It is worth noting that MLS depends on several tuning variables (Table I) and it is difficult to obtain appropriate responses with this algorithm. MLS has the smallest RMS error while Gradient has the biggest (Table II). Average has the smallest settling time while PI the biggest (Table II). Average algorithm takes advantage of sufficient excitation in transient response and estimation of all parameters reach the real value. Due to persistent excitation through function $sin(y)$ of the regressor, estimated mgl reach the real value. For the rest of cases, nonconvergence to real parameters is expected due to the absence of persistent excitation; all that we can expect is stability and convergence to an unknown constant value (Figure 5).

[4]No overshoot present

IV. Conclusion

In this article we presented an unified scheme from which the representative passive AAs Gradient, Composite, Average, PI, and MLS can be derived. A comparative analysis between them was carried out for the problem of adaptive control based on passivity of a simple pendulum. Although performance depends on gains chosen for each law, we tried to put each algorithm in an equilibrated perspective by choosing equivalent constants with equal values. With time varying reference, MLS scheme has the smoothest behavior in tracking error but a big overshoot in control. Due to the presence of persistent excitation through reference signal, all estimated parameters reach real values. With constant reference, MLS has the smoothest behavior in tracking error and control signal. Nonconvergence to real parameters is expected due to the absence of persistent excitation.

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